11.6 ABSOLUTE CONVERGENCE

DEF'N: IF $\sum_{n=1}^{\infty} |a_n|$ CONVERGES,

THEN WE SAY $\sum_{n=1}^{\infty} a_n$ CONVERGES ABSOLUTELY

IF $\sum_{n=1}^{\infty} a_n$ CONV BUT $\sum_{n=1}^{\infty} |a_n|$ DIV,

THEN WE SAY $\sum_{n=1}^{\infty} a_n$ CONV CONDITIONALLY

eg. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONV BY ALT SERIES TEST BUT $\sum_{n=1}^{\infty} \frac{1}{n}$ DIV BY p-SERIES TEST

SO $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONV CONDITIONALLY

eg. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ CONV BY ALT SERIES TEST, AND $\sum_{n=1}^{\infty} \frac{1}{n^2}$ CONV BY p-SERIES TEST

SO $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ CONV ABSOLUTELY

TH'M: IF $\sum_{n=1}^{\infty} a_n$ CONV ABSOLUTELY, THEN $\sum_{n=1}^{\infty} a_n$ CONV

ABSOLUTE CONVERGENCE TEST: • IF $\sum_{n=1}^{\infty} |a_n|$ CONV, THEN $\sum_{n=1}^{\infty} a_n$ CONV

eg. DOES $\sum_{n=1}^{\infty} \frac{\cos n}{n\sqrt{n}}$ CONV/DIV ?

SOME POS, SOME NEG

$\{\cos n\} = \{\cos 1, \cos 2, \cos 3, \cos 4, \ldots\}$

$\neq \{-1, 1, -1, 1, -1, -1, \ldots\}$

CONSIDER $\boxed{\sum_{n=1}^{\infty} \left|\frac{\cos n}{n\sqrt{n}}\right|}$

$-1 < \cos n < 1$

$0 < |\cos n| < 1$

$\frac{0}{n\sqrt{n}} < \frac{|\cos n|}{n\sqrt{n}} < \frac{1}{n\sqrt{n}}$

$\boxed{0 < \left|\frac{\cos n}{n\sqrt{n}}\right| < \frac{1}{n\sqrt{n}}}$

$\downarrow$

$\frac{1}{n^{\frac{3}{2}}}$

$\boxed{\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \text{ CONV}}$ $\boxed{\text{(p-SERIES TEST)}}$

$\boxed{p = \frac{3}{2} > 1}$

$\boxed{\sum_{n=1}^{\infty} \left|\frac{\cos n}{n\sqrt{n}}\right| \text{ CONV}}$ $\boxed{\text{(COMP TEST)}}$

SO, $\boxed{\sum_{n=1}^{\infty} \frac{\cos n}{n\sqrt{n}} \text{ CONV}}$

$\boxed{\text{(ABS CONV TEST)}}$

DOES $\displaystyle\sum_{n=1}^{\infty} \frac{(\sin n)\sqrt{n+1}}{2n^2-1}$ CONV/DIV ?

p-SERIES
GEO
DIV
ALT
LIM COMP
COMP
INT

CONSIDER $\boxed{\displaystyle\sum_{n=1}^{\infty} \left| \frac{(\sin n)\sqrt{n+1}}{2n^2-1} \right|}$

$$\boxed{0 < \left| \frac{(\sin n)\sqrt{n+1}}{2n^2-1} \right| < \frac{\sqrt{n+1}}{2n^2-1}}$$

CONSIDER $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{2n^2-1}$

Σ CONV

$$\frac{\sqrt{n+1}}{2n^2-1} > \frac{\sqrt{n}}{2n^2} = \frac{1}{2}\frac{1}{n^{3/2}}$$

COMPARE TO $\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^{3/2}}}$

$$\boxed{\frac{\sqrt{n+1}}{2n^2-1},\ \frac{1}{2n^{3/2}} > 0}$$

$$\boxed{\lim_{n\to\infty} \frac{\sqrt{n+1}}{2n^2-1} \cdot \frac{2n^2}{\sqrt{n}} = \lim_{n\to\infty} \sqrt{\frac{n+1}{n}} \cdot \frac{2n^2}{2n^2-1}}$$

$$= \lim_{n\to\infty} \sqrt{1+\frac{1}{n}} \cdot \frac{1}{1-\frac{1}{2n^2}}$$

$$= \sqrt{1+0} \cdot \frac{1}{1-0} \boxed{= 1 \neq 0}$$

$$\sum_{n=1}^{\infty} \frac{1}{2n^{3/2}} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$ CONV (p-SERIES)

$p = \frac{3}{2} > 1$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{2n^2-1}$$ CONV (LIM COMP)

$$\sum_{n=1}^{\infty} \left| \frac{(\sin n)\sqrt{n+1}}{2n^2-1} \right|$$ CONV (COMP)

$$\sum_{n=1}^{\infty} \frac{(\sin n)\sqrt{n+1}}{2n^2-1}$$ CONV (ABS CONV)

RATIO TEST

IF $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, THEN $\sum\limits_{n=1}^{\infty} a_n$ CONV

IF $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ OR $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, THEN $\sum\limits_{n=1}^{\infty} a_n$ DIV

IF $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, THEN NO CONCLUSION IS POSSIBLE WITHOUT ADDITIONAL ANALYSIS

eg. CONSIDER $\sum\limits_{n=1}^{\infty} \frac{(-1)^n n!}{e^n}$